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SPIRAL WAVES ON THE SURFACE OF A FILM DRAINING ALONG THE SURFACE OF A VERTICAL CYLINDER

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In this paper, we investigate spiral waves on the surface of a viscous liquid film draining along a vertical tube under the influence of the gravitational force. For this we consider some special solutions of the model equation describing three-dimensional long-wavelength perturbations in such a film for the low flow velocity case.

Perturbations on the surface of a cylindrical film have been investigated earlier, but generally only the case of axially symmetric perturbations has been studied.

Let us consider the flow of a film of viscous liquid along the outer surface of a cylinder of radius R. For any flow velocities, the Navier–Stokes equations have solutions with film thickness h_0 = const. Let us restrict ourselves to the case of large cylinders, for which the relaxation $h_0/R \ll 1$ is valid. In order to make this assumption convenient to use, when writing the equations of motion in a cylindrical coordinate system, instead of the radial coordinate r we will use the new variable (Fig. 1).

$$r' = r - R. \tag{1}$$

In the following, we will omit the prime.

Let V_0 be the characteristic velocity for the non-wave flow regime for a film with thickness h_0, L is the characteristic longitudinal dimension of the perturbation.

Using h_0/V_0, V_0, and $\rho g h_0$ as the time, velocity, and pressure scales and using h_0 and L as the length scales in the r and x directions respectively, let us write the equations of motion for the film in dimensionless form

$$\begin{gathered}
\frac{\partial u}{\partial t} + (V\nabla)u = \frac{1}{\mathrm{Fr}} - \frac{1}{\mathrm{Fr}}\varepsilon\frac{\partial p}{\partial x} + \frac{1}{\mathrm{Re}}\Delta u,\\
\frac{\partial v}{\partial t} + (V\nabla)v - \frac{\delta}{1+\delta r}w^2 = -\frac{1}{\mathrm{Fr}}\frac{\partial p}{\partial r} + \frac{1}{\mathrm{Re}}\left[\Delta v - \frac{\delta^2}{(1+\delta r)^2}v - \frac{2\delta^2}{(1+\delta r)^2}\frac{\partial w}{\partial \varphi}\right],\\
\frac{\partial w}{\partial t} + (V\nabla)w + \frac{\delta}{1+\delta r}vw = -\frac{1}{\mathrm{Fr}}\frac{\delta}{1+\delta r}\frac{\partial p}{\partial \varphi} + \frac{1}{\mathrm{Re}}\left[\Delta w - \frac{\delta^2 w}{(1+\delta r)^2} + \frac{2\delta^2}{(1+\delta r)^2}\frac{\partial v}{\partial r}\right],\\
\frac{\partial v}{\partial r} + \frac{\delta}{1+\delta r}v + \frac{\delta}{1+\delta r}\frac{\partial w}{\partial \varphi} + \varepsilon\frac{\partial u}{\partial x} = 0,
\end{gathered} \tag{2}$$

where

$$V\nabla = v\frac{\partial}{\partial r} + \frac{\delta}{1+\delta r}w\frac{\partial}{\partial \varphi} + \varepsilon u\frac{\partial}{\partial x};$$

$$\Delta = \frac{1}{1+\delta r}\frac{\partial^2}{\partial r^2} + \frac{\delta^2}{(1+\delta r)^2}\frac{\partial^2}{\partial \varphi^2} + \varepsilon^2\frac{\partial^2}{\partial x^2} + \frac{\delta}{1+\delta r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right).$$

The dynamic boundary conditions on the solid (r = 0) and the free boundary respectively have the form

$$u = v = w = 0 \quad (r = 0); \tag{3}$$

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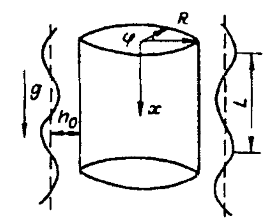

Fig. 1

$$[p - \delta \mathrm{We}(K_1 + K_2)]\frac{n_i}{\mathrm{Fr}} - \sigma_{ik}n_k = 0 \quad (r = h). \tag{4}$$

In (4), the n_i are the components of the unit vector normal to the free surface:

$$(n_r, n_\varphi, n_x) \equiv \left(1, -\frac{\delta}{1 + \delta h}\frac{\partial h}{\partial \varphi}, -\varepsilon\frac{\partial h}{\partial x}\right) \Big/ \left(1 + \frac{\delta^2}{(1 + \delta h)^2}\left(\frac{\partial h}{\partial \varphi}\right)^2 + \varepsilon^2\left(\frac{\partial h}{\partial x}\right)^2\right)^{1/2};$$

the K_i are the dimensionless principal curvatures:

$$K_1 + K_2 = \Big\{(1 + \delta h)^2(1 + \varepsilon^2 h_x^2) - \frac{\varepsilon^2}{\delta}(1 + \delta h)^3 h_{xx} - \delta(1 + \delta h)h_{\varphi\varphi}$$
$$+ 2\delta^2 h_\varphi^2 - \delta\varepsilon^2(1 + \delta h)[h_{xx}h_\varphi^2 - 2h_{\varphi x}h_\varphi h_x + h_{\varphi\varphi}h_x^2]\Big\} \Big/$$
$$\Big/((1 + \delta h)^2(1 + \varepsilon^2 h_x^2) + \delta^2 h_\varphi^2)^{3/2}$$

(here and below, the subscript on h means differentiation with respect to the corresponding variable); the σ_{ik} are the components of the stress tensor:

$$\sigma_{rr} = \frac{2}{\mathrm{Re}}\frac{\partial v}{\partial r}, \quad \sigma_{r\varphi} = \frac{1}{\mathrm{Re}}\left[\frac{\delta}{1 + \delta r}\frac{\partial v}{\partial \varphi} + \frac{\partial v}{\partial r} - \frac{\delta}{1 + \delta r}w\right],$$
$$\sigma_{xr} = \frac{1}{\mathrm{Re}}\left(\frac{\partial u}{\partial r} + \varepsilon\frac{\partial v}{\partial x}\right), \quad \sigma_{\varphi\varphi} = \frac{2}{\mathrm{Re}}\left[\frac{\delta}{1 + \delta r}\frac{\partial w}{\partial \varphi} + \frac{\delta}{1 + \delta r}v\right],$$
$$\sigma_{\varphi x} = \frac{1}{\mathrm{Re}}\left(\varepsilon\frac{\partial w}{\partial x} + \frac{\delta}{1 + \delta r}\frac{\partial u}{\partial x}\right),$$
$$\sigma_{xx} = \frac{2}{\mathrm{Re}}\varepsilon\frac{\partial u}{\partial x}.$$

On the free boundary, the kinematic condition is also satisfied

$$\frac{\partial h}{\partial t} + \frac{\delta}{1 + \delta h}w\frac{\partial h}{\partial \varphi} + \varepsilon u\frac{\partial h}{\partial x} = v \quad (r = h). \tag{5}$$

In (2)-(5), the following appear as parameters: $\varepsilon = h_0/L$, $\delta = h_0/R$, the Reynolds number $\mathrm{Re} = h_0V_0/\nu$, the Froude number $\mathrm{Fr} = V_0^2/gh_0$, the Weber number $\mathrm{We} = \sigma/\rho g h_0^2$. Here ν is the kinematic viscosity coefficient; g is the acceleration of free fall; σ is the surface tension coefficient; ρ is the density of the liquid.

Restricting ourselves to consideration of long-wavelength perturbations, we will assume that $\varepsilon \ll 1$, while the order of δ is no greater than ε, i.e.,

$$\delta = S\varepsilon, \; S \leqslant 1. \tag{6}$$

Let us look for the solution to the problem (2)-(5) in the form of series in ε:

$$u = u_0 + \varepsilon u_1 + \ldots, \quad v = \varepsilon v_1 + \ldots,$$
$$w = \varepsilon w_1 + \ldots, \quad p = \varepsilon p_1 + \ldots \tag{7}$$

Substituting (7) into the original system and equating (taking into account (6)) the coefficients for identical powers of ε to zero, after tedious calculations it is not difficult to obtain expressions for the velocities:

$$u_0 = \frac{\mathrm{Re}}{\mathrm{Fr}}\left(rh - \frac{r^2}{2}\right),$$

$$u_1 = \frac{\mathrm{Re}^3}{\mathrm{Fr}^2} h_x \left[\frac{r^4}{24} h - \frac{r}{6} h^4\right] - \frac{\mathrm{Re}}{\mathrm{Fr}} \mathrm{We}\varepsilon^2 [S^2 h_x + \Delta h_x] \left(\frac{r^2}{2} - hr\right) + S\frac{\mathrm{Re}}{\mathrm{Fr}}\left(\frac{r^3}{6} - \frac{r^2}{2}h + \frac{h^2}{2}r\right) + \frac{\mathrm{Re}^2}{\mathrm{Fr}} h_t \left(\frac{r^3}{6} - \frac{r}{2}h^2\right),$$

$$v_1 = -\frac{\mathrm{Re}}{\mathrm{Fr}}\frac{r^2}{2} h_x,$$

$$v_2 = \frac{\mathrm{Re}}{\mathrm{Fr}} \mathrm{We}\varepsilon^2 \left\{\frac{r^3}{6}(S^2\Delta h + \Delta^2 h) - \frac{r^2}{2}(S^2 h\Delta h + h\Delta^2 h + S^2 h_\varphi \Delta h_\varphi + h_x \Delta h_x + S^4 h_\varphi^2 + S^2 h_x^2)\right\} + S\frac{\mathrm{Re}}{\mathrm{Fr}}\left(\frac{r^3}{3} - \frac{r^2}{2}h\right) h_x$$
$$- \frac{\mathrm{Re}^3}{\mathrm{Fr}^2}\left(h_x^2 \left(\frac{r^5}{120} - \frac{r^2}{3}h^3\right) + h_{xx}\left(\frac{r^5}{120}h - \frac{r^2}{12}h^4\right)\right) - \frac{\mathrm{Re}^2}{\mathrm{Fr}}\left(h_{xt}\left(\frac{r^4}{24} - \frac{r^2}{4}h^2\right) - h_t h_x h \frac{r^2}{2}\right), \tag{8}$$

$$w_1 = S\frac{\mathrm{Re}}{\mathrm{Fr}} \mathrm{We}\varepsilon^2 \left(S^2 h_\varphi + \Delta h_\varphi\right)\left(rh - \frac{r^2}{2}\right).$$

Here $\Delta = \frac{\partial^2}{\partial x^2} + S^2 \frac{\partial^2}{\partial \varphi^2}$. In this case we assumed that the following relations are valid:

$$\mathrm{We}\varepsilon^2 \sim 1, \ \mathrm{Re} \leqslant 1. \tag{9}$$

Substituting (8) into the kinematic condition (5) and restricting ourselves to considering terms up to order ε^2 inclusively, for the perturbed surface h we have

$$h_t + \varepsilon\frac{\mathrm{Re}}{\mathrm{Fr}} h^2 h_x + \varepsilon^2 \left\{S\frac{\mathrm{Re}}{\mathrm{Fr}}\frac{h^3}{3}h_x + \frac{\mathrm{Re}^2}{\mathrm{Fr}}\left[\frac{5}{6}h_t h_x h^3 - \frac{5}{2}h_{xt}h^4\right] - \frac{\mathrm{Re}^3}{\mathrm{Fr}^2}\left[\frac{9}{20}h_x^2 h^5 - \frac{3}{40}h^6 h_{xx}\right] + \frac{\mathrm{Re}}{\mathrm{Fr}}\mathrm{We}\varepsilon^2[(S^2 h_x^2 + h_x \Delta h_x)h^2 + \frac{1}{3}(S^2 h^3 \Delta h + h^3 \Delta^2 h) + S^2 h^2 h_\varphi \Delta h_\varphi + S^4 h_\varphi^2 h^2]\right\} = 0. \tag{10}$$

In the following, let us restrict ourselves to consideration of weakly nonlinear perturbations; therefore we represent the function h in the form

$$h = 1 + \varepsilon H. \tag{11}$$

Using the method of different time scales, let us introduce a set of new variables:

$$t_n = \varepsilon^n t, \ \ n = 1,2, \ldots \tag{12}$$

We note that as the characteristic velocity V_0 we will take the value of the velocity u_0 on the free surface for the non-wave flow regime. From this it follows that

$$\mathrm{Re}/\mathrm{Fr} = 2. \tag{13}$$

Substituting (11) and (12) into (10) and equating the coefficients for identical powers of ε to zero, taking into account (9) and (13), from the first approximation we obtain

$$\frac{\partial H}{\partial t_1} + 2\frac{\partial H}{\partial x} = 0,$$

i.e.,

$$H = H(\xi),\ \xi = x - 2t_1.$$

Considering the second approximation, for determination of H we arrive at the nonlinear equation

$$\frac{\partial H}{\partial t_2} + \frac{2}{3}S\frac{\partial H}{\partial \xi} + 4H\frac{\partial H}{\partial \xi} + \frac{8}{15}\mathrm{Re}\frac{\partial^2 H}{\partial \xi^2} + \frac{2}{3}\mathrm{We}\varepsilon^2(\Delta H S^2 + \Delta^2 H) = 0, \tag{14}$$

where

$$\Delta \equiv \frac{\partial^2}{\partial \xi^2} + S^2\frac{\partial^2}{\partial \varphi^2}.$$

Now it is pertinent to explain more precisely how we choose the longitudinal length scale L. If in (14) we neglect the nonlinear term, then from the linearized equation it follows that the trivial solution $H = 0$ is unstable relative to perturbations of the form

$$\exp(i\alpha(\xi - ct_2) + in\varphi)$$

with components of the wave vector (α, n) satisfying the inequality

$$\frac{8}{15}\mathrm{Re}\alpha^2 + \frac{2}{3}\mathrm{We}\varepsilon^2\{S^2(\alpha^2 + S^2n^2) - (\alpha^2 + S^2n^2)^2\} > 0 \tag{15}$$

(the n are natural numbers, the α are real numbers).

The region of existence of damped perturbations ($c_i < 0$) is determined not by (15) but by an inequality of the opposite sign (<). Accordingly, the wave numbers of neutral perturbations should satisfy the equation

$$\frac{8}{15}\mathrm{Re}\alpha^2 + \frac{2}{3}\mathrm{We}\varepsilon^2\{S^2(\alpha^2 + S^2n^2) - (\alpha^2 + S^2n^2)^2\} = 0. \tag{16}$$

Let us determine the characteristic longitudinal length scale L so that the neutral wave number α_n of axially symmetric perturbations ($n = 0$) will be equal to unity:

$$\alpha_n = 1. \tag{17}$$

Then from (16), taking into account (6), it follows that

$$\varepsilon \equiv \frac{h_0}{L} = \delta(1 + 0.8\mathrm{Re}/\mathrm{We}\delta^2)^{1/2} \tag{18}$$

and accordingly

$$S \equiv \frac{\delta}{\varepsilon} \equiv \frac{L}{R} = 1/(1 + 0.8\mathrm{Re}/\mathrm{We}\delta^2)^{1/2} < 1.$$

The normalization (17) is rather convenient.

Let us introduce a new coordinate

$$\xi_1 = \xi - \frac{2}{3}St_2$$

and let us make the substitution:

$$\tau = \frac{2}{3}\mathrm{We}\varepsilon^2 t_2,\ H = \frac{2}{3}\mathrm{We}\varepsilon^2 H_1. \tag{19}$$

Then Eq. (14), taking into account relation (19), can be rewritten (omitting the subscript on H_1):

$$\frac{\partial H}{\partial \tau} + 4H\frac{\partial H}{\partial \xi} + \frac{\partial^2 H}{\partial \xi^2} + S^4\frac{\partial^2 H}{\partial \varphi^2} + \left(\frac{\partial^2}{\partial \xi^2} + S^2\frac{\partial^2}{\partial \varphi^2}\right)^2 H = 0. \tag{20}$$

Eq. (14) was apparently obtained for the first time in [1]. Of course, in that paper other characteristic scales and accordingly other parameters were used in its derivation and further analysis. For our purposes, the form in (20) is more convenient.

Thus investigation of perturbations in a film draining along the surface of a vertical cylinder, within the constraints used, is reduced to analysis of the solutions of Eq. (20). In the case of axially symmetric solutions ($H = H(\tau, \xi)$), it is transformed to the well known equation which often is called the Kuramoto–Sivashinsky equation:

$$\frac{\partial H}{\partial \tau} + 4H\frac{\partial H}{\partial \xi} + \frac{\partial^2 H}{\partial \xi^2} + \frac{\partial^4 H}{\partial \xi^4} = 0 \tag{21}$$

(strictly speaking, there is a difference between the nonlinear terms in the Kuramoto–Sivashinsky equation and (21)).

When going in the limit from a cylindrical to a flat surface, $S \to 0$. In this case introducing, instead of the coordinate φ, the new coordinate $z = R\varphi/L$, from Eq. (20) we go to

$$\frac{\partial H}{\partial \tau} + 4H\frac{\partial H}{\partial \xi} + \frac{\partial^2 H}{\partial \xi^2} + \left(\frac{\partial^2}{\partial \xi^2} + \frac{\partial^2}{\partial z^2}\right)^2 H = 0. \tag{22}$$

Equation (22) was obtained in [2] in an investigation of three-dimensional perturbations on a vertical flat film.

In this paper, let us restrict ourselves to looking for solutions of the spiral type for Eq. (20), i.e., solutions for which the following is valid:

$$H = H(\tau, \eta),\ \eta = \xi + \gamma\varphi. \tag{23}$$

Since φ is the angular coordinate, any solution of Eq. (20) should be periodic with respect to φ, with period fitting into the interval 0-2π an integral number of times ($n \geq 1$). Therefore it is clear that if solutions of type (23) exist for Eq. (20), then they should be periodic also with respect to the variable η.

Let the solution (23) exist and have period λ. Clearly the period with respect to the variable ξ also will be λ. In this case, as has already been noted, the period with respect to the variable φ should be equal to $2\pi/n$. It is easy to understand that, in order for this to be possible, the value of γ in (23) should not be arbitrary, but rather should satisfy the relation

$$\lambda = 2\pi\gamma/n, \quad \text{or} \quad n/\gamma = \alpha \tag{24}$$

(α is the wave number).

In order to determine solutions of the spiral type (23) for Eq. (20), we use the substitution:

$$\begin{gathered} H(\tau, \eta) = AH_1(\tau_1, \eta_1), \\ \tau_1 = \varkappa\tau,\ \eta_1 = p\eta, \\ p = (1 + S^4\gamma^2)^{1/2}/(1 + S^2\gamma^2),\ \varkappa = p^2(1 + S^4\gamma^2),\ A = \varkappa/p. \end{gathered} \tag{25}$$

Then for H_1, Eq. (20) is rewritten as follows:

$$\frac{\partial H_1}{\partial \tau_1} + 4H_1\frac{\partial H_1}{\partial \eta_1} + \frac{\partial^2 H_1}{\partial \eta_1^2} + \frac{\partial^4 H_1}{\partial \eta_1^4} = 0 \tag{26}$$

(i.e., for the function H_1 we have Eq. (21)).

Equation (26) at present is the classical equation. Often we arrive at this equation when describing wave processes in different actively dissipative media. We know that its solutions also include periodic traveling-wave solutions. If for some periodic solution H_1 the relations (24) and (26) can be satisfied, then the solution H connected with it from (23) will also be the corresponding spiral solution to Eq. (20).

Let α_x be the wave number for the solution H, while α_ξ is the wave number for the solution H_1. It is easy to show that in order for relations (24) and (25) to be satisfied, these wave numbers should satisfy the equation

$$\alpha_\xi^2(\alpha_x^2 + S^4n^2) = (\alpha_x^2 + S^2n^2)^2. \tag{27}$$

Thus the problem is reduced to determining the unknown wave number α_x, using formula (27) for the known solution H_1 with wave number α_ξ. The corresponding solution H is then determined from formulas (23) and (25). Since in (27) n is present only in the odd powers, it is clear that if Eq. (27) has a solution α_x, then it is the same for $\pm$n. This means that if a spiral solution exists with a left-handed twist (as we move along a level of constant phase toward an increase in the coordinate ξ, rotation with respect to the coordinate φ is accomplished in a counterclockwise direction), then there is also a solution symmetric to it with a right-handed twist. Therefore in the following, we will not talk about solutions with $n < 0$.

It is well known (see, for example, [3-6]) that Eq. (26) has a stationary traveling-wave solution of the form

$$H_1 = H_1(\eta_1 - c\tau_1). \tag{28}$$

The solutions to (28) include a countable set of periodic and soliton solutions. As is easily seen from (27), no spiral solutions corresponding to soliton solutions for H_1 exist.

The wave numbers α_ξ of the entire set of periodic solutions (28) lies in the interval 0-1. This set is divided into two classes. The first class includes solutions with values of the phase velocity $c = 0$. They all are antisymmetric. Among them there exists one family of solutions (traditionally called the first family) which branches with infinitesimally small amplitude from the trivial solution for $\alpha_\xi = 1$ and continuous to the wave number $\alpha_\xi = 0.4979$. For waves of this family, there is an interval of wave numbers ($0.77 < \alpha_\xi < 0.84$) stable to all perturbations. Wave numbers of the other antisymmetric families are less than 0.42.

For solutions of the second type, the values of the phase velocity $c \neq 0$ (and by virtue of the symmetry of Eq. (26), it is sufficient to talk about only solutions with $c > 0$). The wave numbers for these waves lie in the interval 0-0.554, In the limit when the wave number goes to zero, these periodic solutions go to solitary waves.

For solutions of (25) corresponding to the antisymmetric solutions, the surface of the film is represented by spiral cords in which the hills are similar to the valleys. For solutions with $c > 0$, there is no such similarity. The possible number of cords n and the step of such a spiral wave $\lambda = 2\pi/\alpha_x$ depend on the values of the parameter S. In fact, let us write out explicitly the roots of the quadratic equation (27):

$$\alpha_{x_{1,2}}^2 = \{\alpha_\xi^2 - 2S^2n^2 \pm [(\alpha_\xi^2 - 2S^2n^2)^2 - 4S^4n^2(n^2 - \alpha_\xi^2)]^{1/2}\}/2. \tag{29}$$

Since we need the values of α_ξ to satisfy the inequality

$$0 \leqslant \alpha_\xi \leqslant 1,$$

clearly if these roots are real then they are either both negative or both positive. In the latter case, we have two different wave numbers α_x corresponding to the same α_ξ, i.e., one "flat" solution (26) "generates" two stationary traveling spiral solutions (23) (with the same $n > 0$), differing in wavelengths and slope of the spirals. For θ (the angle between the axis of the longitudinal coordinates ξ and the lines of constant phase η_1 = const), the following relation is valid:

$$\mathrm{tg}\theta = \alpha_x/n.$$

From (29) it is easy to see that for fixed values of n and S, the value of α_{x_1} will be the greatest while the value of α_{x_2} will be the smallest for $\alpha_\xi = 1$. On the (α_x, S) plane, these values outline the region of existence of the spiral solutions with the corresponding azimuthal number n. In Fig. 2, this region is found within the curve 1. here a, b correspond to $n = 1$ and 2. As we see from (29) and Fig. 2a, for $n = 1$ the minimum possible wave number of the spiral wave is $\alpha_x = 0$, i.e., for any S for which such solutions exist, we can construct a spiral solution with as large a step as desired along the longitudinal coordinate . For any $n \geq 2$ and any allowable S, the spiral solution cannot have an angle θ less than some finite angle.

The interval of allowable values of S for these solutions is determined by the inequality

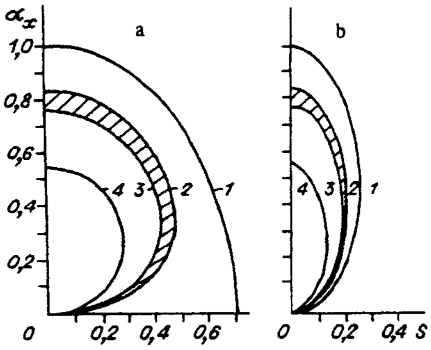

Fig. 2

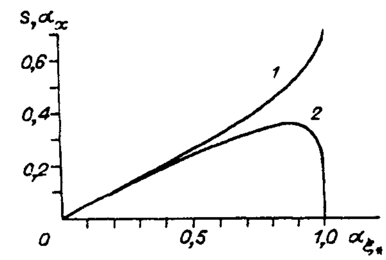

Fig. 3

$$S^2 \leqslant S_*^2 = [n - (n^2 - 1)^{1/2}]/2n. \tag{30}$$

For S*, when $\alpha_\xi = 1$ we have

$$\alpha_{x_1} = \alpha_{x_2} = [(1 - S^2n^2)/2]^{1/2}.$$

From (30) it follows that for sufficiently large values of S ($S > S_1 = 1/\sqrt{2}$), all spiral wave regimes are forbidden. As we see from (6) and (18), this is the case of rather "small" cylinders. For values of $S > S_2 = S_1(1 - \sqrt{3/2})^{1/2}$, regimes with number of cords $n > 1$ are forbidden (i.e., for "moderately" large cylinders, spiral regimes are allowed only with period 2π along the angular coordinate) etc. With a decrease in S, both the interval of possible wave numbers for such regimes and also the allowed number of their cords increase.

If for a given value of S there is a finite interval of allowable wave numbers α_x, then (as is easy to understand from (29)) there exists $\alpha_{\xi_*} < 1$, for which $\alpha_{x_1} = \alpha_{x_2}$. This means that the wave numbers of the corresponding flat solution (26) lie in the interval $\alpha_{\xi_*} \leq \alpha_\xi \leq 1$. This is explained by the curves in Fig. 2. They all represent lines α_ξ = const, and specifically: curves 1-4 correspond to α_ξ = 1, 0.84, 0.77, 0.554.

Knowing the dependence $S = S(\alpha_{\xi_*})$, we can determine in particular the families of each type of all the possible solutions to (26) that can be generated by the corresponding spiral solutions (23). Thus in Fig. 3, such a dependence is represented by curve 1 for n = 1; the dependence $\alpha_x = \alpha_x(S(\alpha_{\xi_*}), \alpha_{\xi_*})$ is shown by curve 2. From these data it is easy to understand, for example, that only the first family of solutions to (26) will give spiral solutions (this will be for those values of S for which $\alpha_{\xi_*} \geq 0.554$), for which values of S stable spiral regimes are possible ($0.77 \leq \alpha_{\xi_*} \leq 0.84$), etc.

In Fig. 2, the region of stability is shown by the hatching. We see that for values of $S < S_+ = S(\alpha_{\xi_*} = 0.77)$, there are two regions of wave numbers α_x stable to all spiral perturbations. These regions merge into one for $S \geq S_+$.

The region of existence of spiral solutions having phase velocity grater than the velocity of infinitesimally small perturbations (solutions to which correspond the solutions of (26) with $c > 0$) is considerably narrower. It is shown in Fig. 2 by the curve 4.

Thus, by using relations (23)-(27), we can obtain considerable information about the spiral wave regimes on the surface of vertical cylinders. Considering that the parameter S, according to (6) and (18), represents a combination of geometric and dynamic factors, within the framework of this model we obtain in particular a qualitatively reasonable result: for fixed values of the Reynolds number, the wave pattern of possible spiral waves is more abundant as we consider larger cylinders (S decreases with an increase in the radius of the cylinder R). Such a conclusion is valid if for a fixed cylinder we increase the flow velocity of the liquid.

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